Multigluon correlations and initial state azimuthal anisotropies in the glasma

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UNIVERSITY OF JYVÄSKYLÄ

HELSINKI INSTITUTE OF PHYSICS

Outline

- CGC and glasma
- Dilute-dense collisions, Wilson line in target color field
- Is it always flow?
- Calculating anisotropies in dilute-dense system "pA"
- Calculating gluon production in dense-dense "AA"

Gluon saturation, Glass and Glasma

Small x: the hadron/nucleus wavefunction is characterized by **saturation scale** $Q_s \gg \Lambda_{QCD}$.

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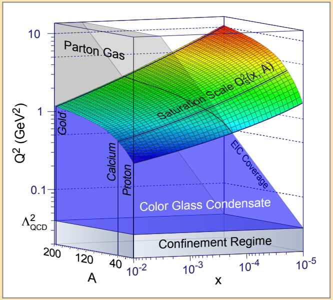
▼

$\mathbf{p}_T \sim Q_s$: strong fields $A_\mu \sim 1/g$

- occupation numbers $\sim 1/\alpha_s$
- classical field approximation.
- small α_s, but nonperturbative



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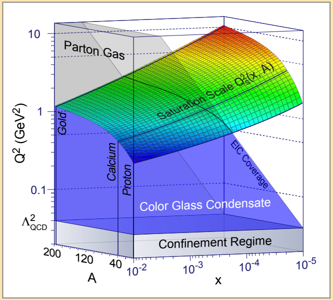

CGC: Effective theory for wavefunction of nucleus

- Large x = source ρ, **probability** distribution $W_y[\rho]$
- Small x = classical gluon field A_μ + quantum flucts.

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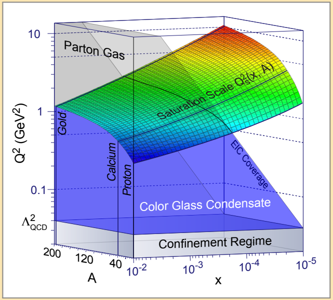

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Glasma: field configuration of two colliding sheets of CGC.
JIMWLK: y-dependence of $W_y[\rho]$; Langevin implementation

Wilson line

Classical color field described as Wilson line

In practice degree of freedom is not ρ but Wilson line:

$$V(\mathbf{x}_T) = P\exp\left\{ig\int \mathrm{d}x^- A^+_{\mathrm{cov}}(\mathbf{x}_T, x^-)\right\} \quad \in \quad \mathrm{SU}(3)$$

$$\text{Color charge } \rho: \quad {\nabla_T}^2 A^+_{\mathrm{cov}}(\mathbf{x}_T, x^-) = -g\rho(\mathbf{x}_T, x^-)$$

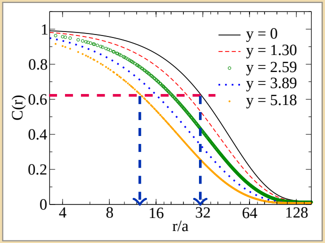
($x^\pm = \frac{1}{\sqrt{2}}(t \pm z)$; $A^\pm = \frac{1}{\sqrt{2}}(A^0 \pm A^z)$; $\mathbf{x}_T$ 2d transverse)

Q_s is characteristic momentum/distance scale

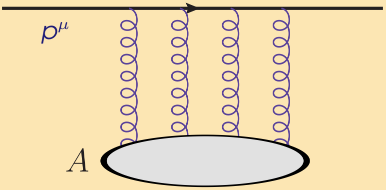
Precise definition used here:

$$C(\mathbf{x}_T) = \frac{1}{N_c}\left\langle \mathrm{Tr}\, V^\dagger(\mathbf{0}_T) V(\mathbf{x}_T)\right\rangle = e^{-\frac{1}{2}}$$

$$\iff {\mathbf{x}_T}^2 = \frac{2}{Q_s^2}$$



Why Wilson line



Quark propagating in classical color field: Dirac equation!

$$(i\partial\!\!\!/ - gA\!\!\!/)\psi(x) = 0$$

(Note: $A\!\!\!/ = A_a^\mu \gamma_\mu t^a$ is $N_c \times N_c$-matrix; ψ is a vector with $4N_c$ components)

Dominant high energy contribution: **eikonal** approximation

- Gluon is spin 1: it couples to a vector: $\sim p^\mu A_\mu$
- For high energy particle the only momentum available is p^μ
- p^μ has one large component: p^+

$\Longrightarrow p^\mu A_\mu \sim p^+ A^-$ $\Longrightarrow$ only need A^-

Ansatz for DE: $\psi(x) = V(x) e^{-ip\cdot x} u(p)$, plug in: $N_c \times N_c$**-matrix!**

$$\Longrightarrow \quad \partial_+ V(x^+, x^-, \mathbf{x}_T) = -igA^-(x^+, x^-, \mathbf{x}_T) V(x^+, x^-, \mathbf{x}_T)$$

This is solved by path-ordered exponential

$$V(x^+, x^-, \mathbf{x}_T) = \mathbb{P} \exp\left\{ -ig \int^{x^+} \mathrm{d}y^+ A^-(y^+, x^-, \mathbf{x}_T) \right\}$$

At H.E. neglect x^-: any dependence is weak compared to $e^{-ip^+x^-}$

JIMWLK evolution

Classical color field described as Wilson line

$$V(\mathbf{x}_T) = P\exp\left\{ig\int \mathrm{d}x^- A^+(\mathbf{x}_T, x^-)\right\} \in \mathrm{SU}(3)$$

- Energy dependent **probability** distribution $W_y[V]$ $(y \sim \ln\sqrt{s})$
- Energy/rapidity dependence of $W_y[V]$ given by JIMWLK renormalization group equation

$$\partial_y W_y[V(\mathbf{x}_T)] = \mathcal{H} W_y[V(\mathbf{x}_T)]$$

- Then get all expectation values $\langle V \cdots V^\dagger \rangle$

JIMWLK Hamiltonian: (fixed coupling)

$$\mathcal{H} \equiv \frac{1}{2}\alpha_s \int_{\mathbf{x}_T\mathbf{y}_T\mathbf{z}_T} \frac{\delta}{\delta A_c^+(\mathbf{y}_T)} \mathbf{e}_T^{ba}(\mathbf{x}_T, \mathbf{z}_T) \cdot \mathbf{e}_T^{ca}(\mathbf{y}_T, \mathbf{z}_T) \frac{\delta}{\delta A_b^+(\mathbf{x}_T)},$$

$$\mathbf{e}_T^{ba}(\mathbf{x}_T, \mathbf{z}_T) = \frac{1}{\sqrt{4\pi^3}} \frac{\mathbf{x}_T - \mathbf{z}_T}{(\mathbf{x}_T - \mathbf{z}_T)^2} \left(1 - U^\dagger(\mathbf{x}_T) U(\mathbf{z}_T)\right)^{ba}$$

(Here U is adjoint reps of V)

JIMWLK in Langevin formulation

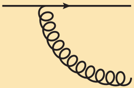
Fokker-Planck $\Longrightarrow$ Langevin in JIMWLK Blaizot, Iancu, Weigert 2002

Simple form for Langevin step

$$V_{\mathbf{x}_T}(y+\mathrm{d}y) = \exp\left\{-i\frac{\sqrt{\mathrm{d}y}}{\pi}\int_{\mathbf{z}_T} \mathbf{K}_{T\,\mathbf{x}_T-\mathbf{z}_T}\cdot\left(V_{\mathbf{z}_T}\xi_{T\,\mathbf{z}_T}V^\dagger_{\mathbf{z}_T}\right)\right\}$$

$$\times V_{\mathbf{x}_T}(y)\exp\left\{i\frac{\sqrt{\mathrm{d}y}}{\pi}\int_{\mathbf{z}_T}\mathbf{K}_{T\,\mathbf{x}_T-\mathbf{z}_T}\cdot\xi_{T\,\mathbf{z}_T}\right\},$$

$$K^i_{\mathbf{x}_T-\mathbf{z}_T} = \frac{(\mathbf{x}_T-\mathbf{z}_T)^i}{(\mathbf{x}_T-\mathbf{z}_T)^2}$$



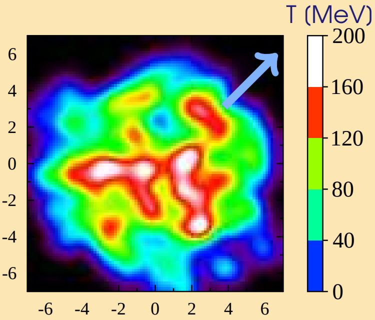
$i = x, y$

Noise: $\langle \xi_{\mathbf{x}_T}(y_m)^a_i\,\xi_{\mathbf{y}_T}(y_n)^b_j\rangle = \alpha_s\delta^{ab}\delta^{ij}\delta^{(2)}_{\mathbf{x}_T\mathbf{y}_T}\delta_{mn}, \quad \xi = \xi^a t^a$

More recent developments not discussed here:

- Fixed $\Longrightarrow$ running α_s: proposal by T.L., Mäntysaari 2012
- Full NLO Balitsky, Chirilli 2013, Kovner, Lublinsky, Mulian 2014
  - NLO BFKL/BK problematic, treatment for JIMWLK not obvious.

Flow in hydro



- Interactions/collectivity
- \+ Temperature/pressure gradients

⟹ Anisotropic force, acceleration
→ anisotropy in momentum

Large system:

⟹ details of MC Glauber matter little

⟹ initial geometry under control

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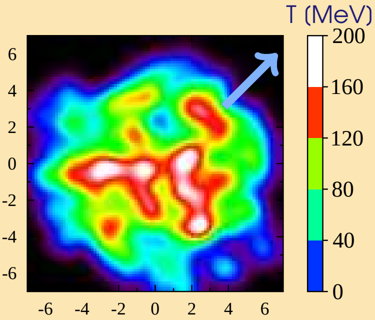

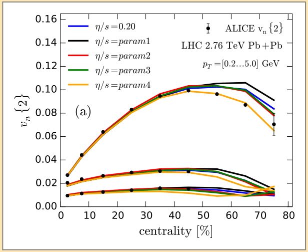

Works well in AA Niemi et al

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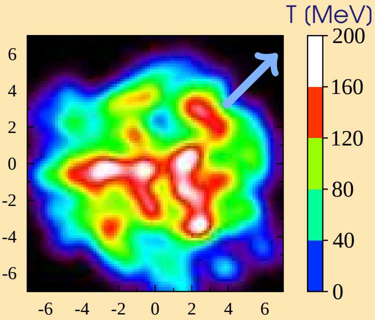

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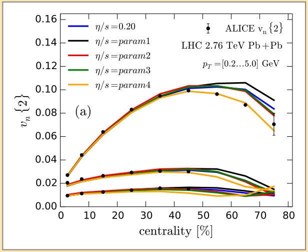

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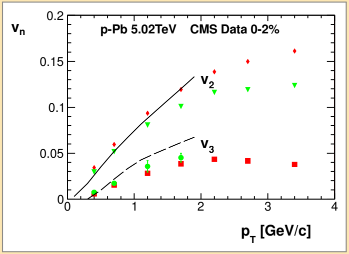
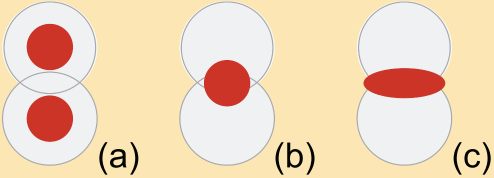


But also in small systems? Bozek, Broniowski

Flow in small systems

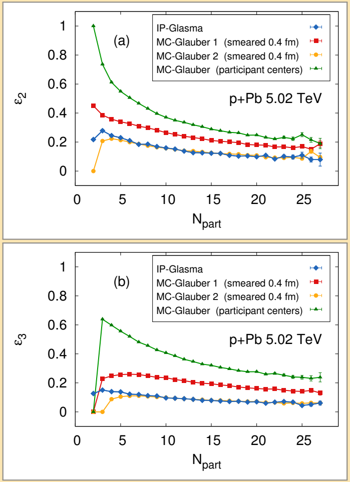
Want to do MC Glauber for pA.
How is the energy distributed?



Eccentricities very model-dependent

Therefore: "hydro prediction for flow" has large initial state theory uncertainty in small systems.

Hydro predictions for v_n in pp
at similar N_{ch}?



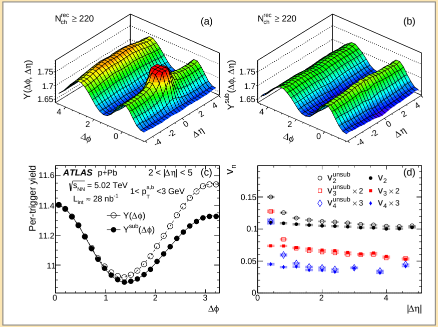
(Green: (a), red: (a) smeared, yellow: (b) smeared)

Bzdak, Schenke, Tribedy Venugopalan,

Long range in rapidity: early time

- Long range rapidity correlations: early time
  - Analogous to CMB
- v_n= multiparticle correlation

  (usually long range in rapidity)
- Geometry is the ultimate infinite-range correlation
  - All rapidities sensitive to ⊥ geometry
  - Hydro translates *x*-space correlations into *p*-space

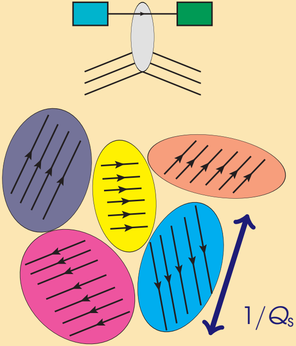


Seen as yield/trigger or as v_n: ATLAS, Phys. Rev. C **90** (2014) 4, 044906 [arXiv:1409.1792 [hep-ex]].

Initial state QCD long range effects:
non-geometry correlations directly in momentum space

Domains in the target color field

Initial state CGC correlations in dilute-dense limit



- $\sim$collinear high-x q/g
- Momentum transfer from target E-field
- Domains of size $\sim 1/Q_s$
- Several particle see same domain: multiparticle azimuthal correlations.

- $\sim Q_s^2 S_\perp$ domains ($S_\perp$ = size of interaction area, πR_A^2, πR_p^2)
- $\sim {N_c}^2$ colors

Correlation $\frac{1}{{N_c}^2 Q_s^2 S_\perp}$ $\Longrightarrow$ relatively stronger in small systems

Explicit setup for dilute-dense

TL Phys. Lett. B **744** (2015) 315 (arXiv:1501.05505 (hep-ph))

- Passage of probe particle through color field: eikonal Wilson line in target color field

$$V(\mathbf{x}_T) = P\exp\left\{ig\int \mathrm{d}x^- A^+_{\mathrm{cov}}(\mathbf{x}_T, x^-)\right\}$$

- Localize quarks in Gaussian wave packet in probe:

$$\frac{\mathrm{d}N}{\mathrm{d}^2\mathbf{p}_T} \propto \int_{\mathbf{x}_T,\mathbf{y}_T} e^{-i\mathbf{p}_T\cdot(\mathbf{x}_T-\mathbf{y}_T)} e^{\frac{-(\mathbf{x}_T-\mathbf{b}_T)^2}{2B}} e^{\frac{-(\mathbf{y}_T-\mathbf{b}_T)^2}{2B}} \frac{1}{N_c}\,\mathrm{Tr}\, V^\dagger_{\mathbf{x}_T} V_{\mathbf{y}_T}.$$

- Two particle correlation

$$\frac{\mathrm{d}N}{\mathrm{d}^2\mathbf{p}_T\,\mathrm{d}^2\mathbf{q}_T} = \int \dots \left\langle \frac{1}{N_c}\,\mathrm{Tr}\, V^\dagger_{\mathbf{x}_T} V_{\mathbf{y}_T} \frac{1}{N_c}\,\mathrm{Tr}\, V^\dagger_{\mathbf{u}_T} V_{\mathbf{v}_T}\right\rangle \quad \Longrightarrow \quad v_n\{2\}$$

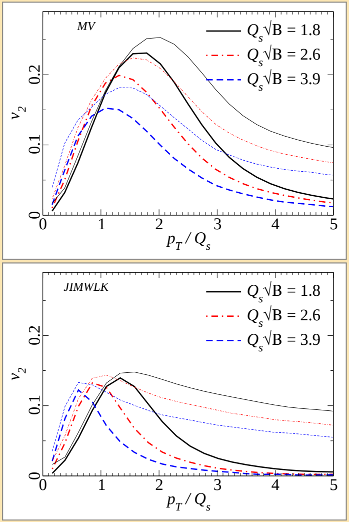
- Need distribution of Wilson lines V for Monte Carlo: MV or JIMWLK (in Langevin method)

Anisotropy coefficients from JIMWLK and MV

TL Phys. Lett. B **744** (2015) 315 (arXiv:1501.05505 (hep-ph))

- p_T-structure like data, but peak at lower p_T
- Depends on probe size B
- Stronger for larger x (MV)

v_2



- Thick line: correlate p_T vs all
- Thin line: p_T vs p_T

Target homogenous & isotropic
$\Longrightarrow$ v_n from fluctuations, not geometry

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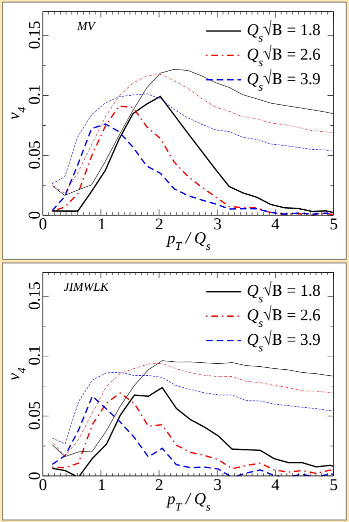
v_4



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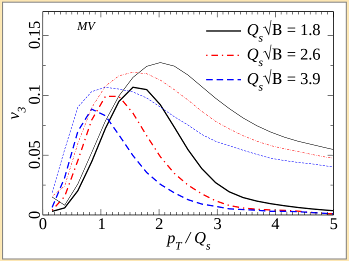
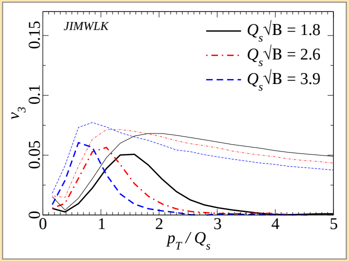
- v_4 at higher p_T
- Odd v_n only for quark probe

v_3





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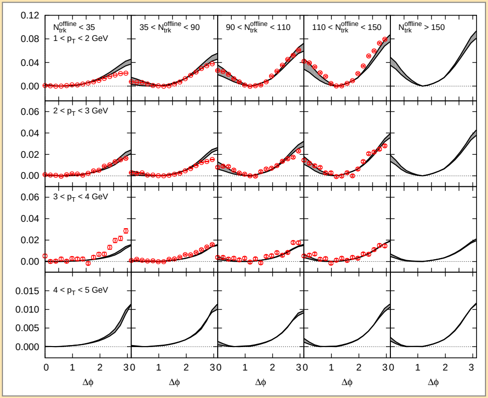
Calculations in the literature

Azimuthal correlations analyzed in terms of the

- “Glasma graph” ridge correlation



Dusling, Venugopalan, Phys. Rev. D **87** (2013) 9, 094034 [arXiv:1302.7018 [hep-ph]].

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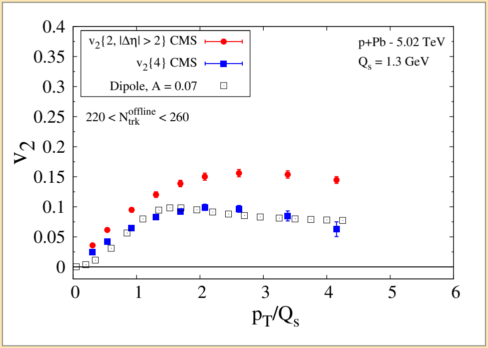

Dumitru, Giannini, Nucl. Phys. A **933** (2014) 212 [arXiv:1406.5781 [hep-ph]].

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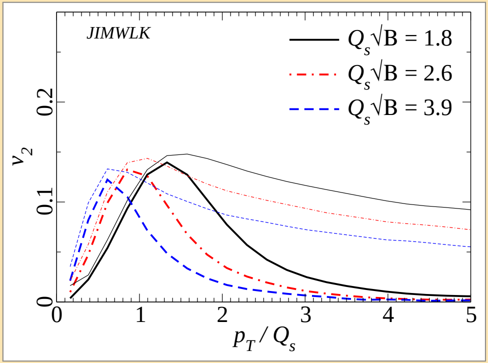

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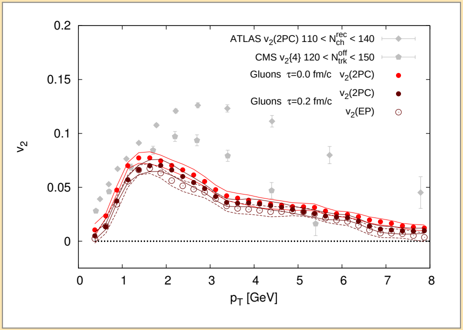

Schenke, Schlichting, Venugopalan, Phys. Lett. B **747** (2015) 76 [arXiv:1502.01331 [hep-ph]].

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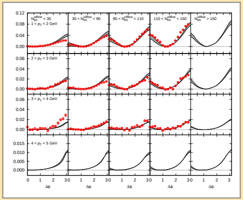
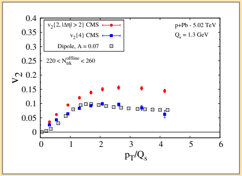
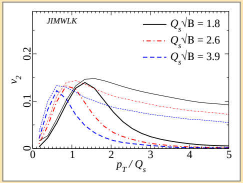
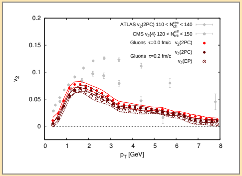








Physics of color field domains same; approximations different

Difference between approximations

For $V(\mathbf{x}_T) = P \exp \left\{ ig \int \mathrm{d}x^- \frac{\rho(\mathbf{x}_T, x^-)}{\boldsymbol{\nabla}_T{}^2} \right\}$,

need $\left\langle \mathrm{Tr}\, V^\dagger(\mathbf{x}_T) V(\mathbf{y}_T)\, \mathrm{Tr}\, V^\dagger(\mathbf{u}_T) V(\mathbf{v}_T) \right\rangle$

Approximations in dilute-dense

- JIMWLK: Langevin equation for $V(\mathbf{x}_T)$.
  Close to Gaussian in ρ, but nonlinear ("nonlinear Gaussian")
- "Glasma graph": linearize in ρ, Gaussian ρ
- "E-field domain model", small dipole limit

$$\frac{1}{N_c} V^\dagger(\mathbf{b}_T + \mathbf{r}_T/2) V(\mathbf{b}_T - \mathbf{r}_T/2) \approx 1 - \frac{r^i r^j}{4N_c} E_i^a(\mathbf{b}_T) E_j^a(\mathbf{b}_T)$$

+ non-Gaussian 4-point correlation with extra parameter $\mathcal{A}$

CYM: nonlinear with Gaussian ρ for **both** nuclei

+ final state evolution

Approximations for Wilson line correlator

T. L., B. Schenke, S. Schlichting and R. Venugopalan, arXiv:1509.03499 [hep-ph]

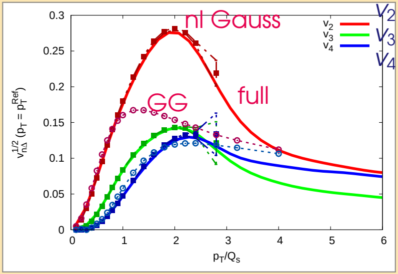
Compare full MV or JIMWLK $v_n\{2\}$ to

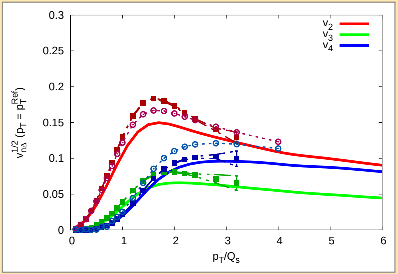
- Nonlinear Gaussian (Gaussian ρ, do not linearize) :

  accurate within 10%

- "Glasma graph" (Gaussian + linearized)

  differs by factor 2 at most

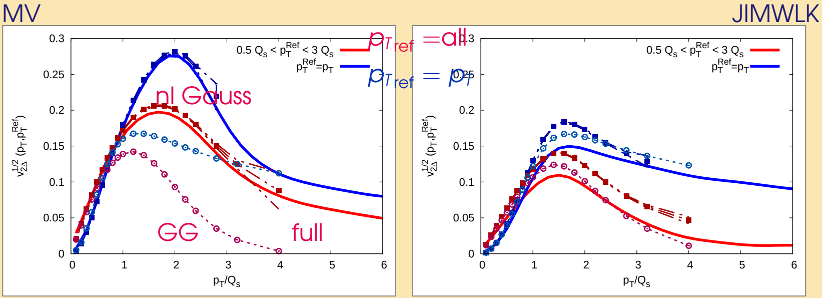




MV JIMWLK

Remarkable consistency between approximations

Effect of reference p_T



- MV
  - Correlation more localized in p_T than experimental data (Hadronization will change this, but how much?)
  - GG decorrelates paricularly fast
- JIMWLK:
  - Little difference between approximations

Color field domain model

A. Dumitru and A. V. Giannini, Nucl. Phys. A **933** (2014) 212 [arXiv:1406.5781 [hep-ph]]

$$\left\langle E^i E^j \right\rangle \sim \left[\delta^{ij}(1-\mathcal{A}) + 2\mathcal{A}\hat{a}^i\hat{a}^j\right]$$

Then average over color field direction $\hat{a}$.
Result: non-Gaussianity with unknown parameter $\mathcal{A}$:

$$\langle EEEE \rangle = \Big(\overbrace{3}^{\text{Gaussian}} + \overbrace{\mathcal{A}^2}^{\text{from } \hat{a}} \Big) \langle EE \rangle \langle EE \rangle$$

What does $\mathcal{A}$ represent?

1. Effect of nonlinearities?
   "Glasma graph" linearization is factor $\sim$2 effect.
2. Nongaussianities from JIMWLK?
   $\sim$10% effect, but interesting for theorist.
3. New structure beyond conventional CGC (MV+JIMWLK)?
   Origin? Timescales? N_c-counting?

For the future: rapidity structure

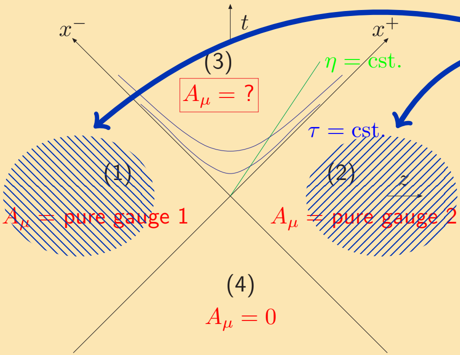
- All of these neglect decorrelation in rapidity due to gluon emissions, parametrically true only for $\Delta y \lesssim 1/\alpha_s$
- Rapidity decorrelation formulated

  Iancu, Triantafyllopoulos, JHEP **1311** (2013) 067 [arXiv:1307.1559 [hep-ph]]

  but not implemented

Gluon fields in AA collision

Classical **Y**ang-**M**ills



Change to LC gauge:

$$A^i_{(1,2)} = \frac{i}{g} U_{(1,2)}(\mathbf{x}_T)\partial_i U^\dagger_{(1,2)}(\mathbf{x}_T)$$

$U(\mathbf{x}_T)$ is the same Wilson line

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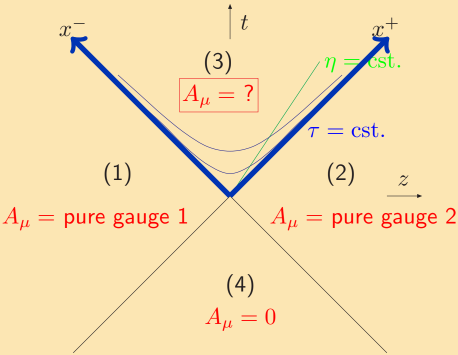

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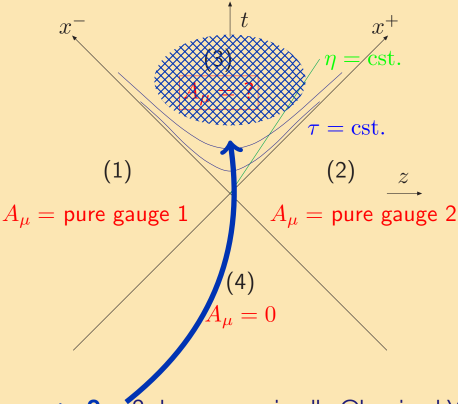

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$\tau > 0$ Solve numerically Classical Yang-Mills **CYM** equations.
This is the **glasma** field $\Longrightarrow$ Then average over initial Wilson lines.

Fix gauge, Fourier-decompose: gluon spectrum

Gluons with $p_T \sim Q_s$ — strings of size $R \sim 1/Q_s$

Gluon spectrum in the glasma

T.L., *Phys.Lett.* **B703** (2011) 325

Q_s is only dominant scale

Parametrically gluon spectrum $$\frac{dN_g}{dy\, d^2\mathbf{x}_T\, d^2\mathbf{p}_T} = \frac{1}{\alpha_s} f\left(\frac{p_T}{Q_s}\right)$$

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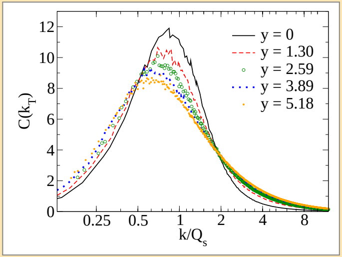

Unintegrated gluon distribution

$$C(\mathbf{k}_T) = \frac{{k_T}^2}{N_c} \mathrm{Tr} \left\langle U(\mathbf{k}_T) U^\dagger(\mathbf{k}_T) \right\rangle$$

becomes **harder** with evolution.

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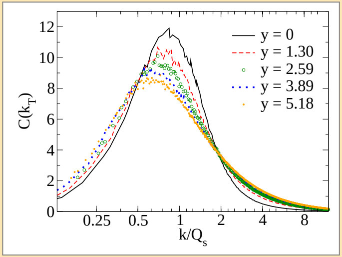

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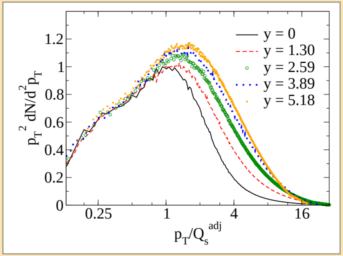

Produced gluon spectrum: harder at higher $\sqrt{s}$

(Here: midrapidity, $y \equiv \ln \sqrt{s/s_0}$)

Glittering Glasma

Correlations simple in MV model and dilute limit (small ρ)

$$W[\rho] = \exp[- \int d^2\mathbf{x}_T \frac{\rho^a(\mathbf{x}_T)\rho^a(\mathbf{x}_T)}{g^4\mu^2})$$

- 2-particle Dumitru, Gelis, McLerran, Venugopalan arXiv:0804.3858
- 3-particle Dusling, Fernandez-Fraile, Venugopalan arXiv:0902.4435
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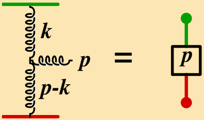
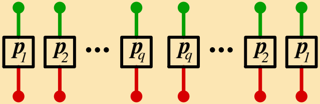
Building block:



Connect dots in



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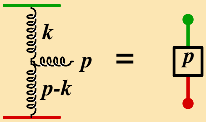
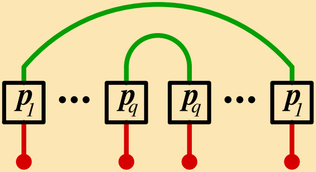


Dominant: **glasma graphs**



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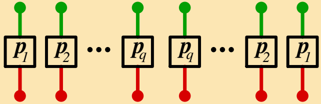

Result

Number of diagrams $\sim (q-1)!$

Negative binomial

Moment $m_q \equiv \langle N^q \rangle$ – disc.

$$
\begin{aligned}
m_q &= (q-1)!\, k \left(\frac{\bar{n}}{k}\right)^q \\
k &\approx \frac{({N_c}^2-1) Q_s^2 S_\perp}{2\pi} \\
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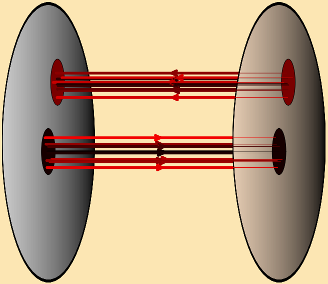
Flux tube interpretation



$Q_s^2 S_\perp = N_{FT}$ # of flux tubes
$k \approx N_{FT}({N_c}^2 - 1) =$ emitters
Each emitter produces particles with Bose-Einstein distribution

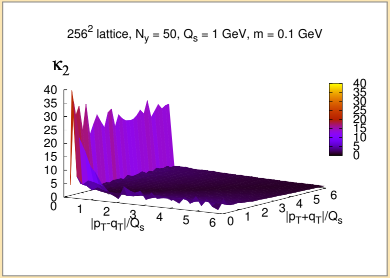
(Negative binomial is sum of k independent BE's)

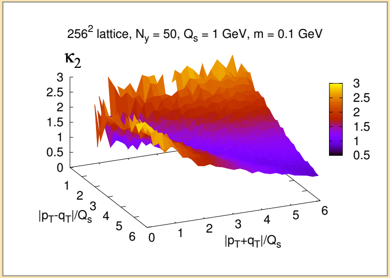
Boost invariant correlation: full numerical calculation

T.L., Srednyak, Venugopalan, arXiv:0911.2068

$$\kappa_2(\mathbf{p}_T, \mathbf{q}_T) = \overbrace{S_\perp Q_s^2}^{\text{\# of independent regions}} \frac{\left\langle \frac{d^2 N_2}{dy_p\, d^2\mathbf{p}_T\, dy_q\, d^2\mathbf{q}_T} \right\rangle}{\left\langle \frac{dN}{dy_p\, d^2\mathbf{p}_T} \right\rangle \left\langle \frac{dN}{dy_q d^2\mathbf{q}_T} \right\rangle} - 1$$

Dilute limit: $\kappa_2(\mathbf{p}_T, \mathbf{q}_T) \sim 1/(N_c{}^2 - 1)$ constant up to logs.





Conclusions

- Strong multiparticle azimuthal correlations seen even in small systems
- Interpretation as initial vs. final state collectivity still open
- Initial gluon field can be a significant source of correlation
  - Especially for small systems
  - Hadronization, p_T-dependence?